Characteristics of Waveguides with a Semiconductor Side Wall

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Abstract—**The attenuation, guide wavelength, and characteristic impedance of rectangular waveguides with one high conductivity semiconductor side wall have been derived for the case of propagating TE_{N0} modes. These properties can be interpreted in terms of the penetration of the microwave electric field into the semiconductor material by an amount of the same order as, but generally unequal to, the classical skin depth. These theoretical results are evaluated for the special case of an indium antimonide side wall in RG 138/U waveguide operating at 110 Gc/s. The calculated attenuation lengths and guide wavelengths for this case are of such magnitude that they can be measured with reasonable accuracy, thus illustrating the value of this technique for the measurement of the electrical properties of semiconductor materials at the higher microwave frequencies.**

Introduction

THERE HAS BEEN an increasing interest in the microwave properties of semiconductor and semimetal materials. This interest has been expressed in theoretical studies [1]–[6] as well as in experimental investigations [7]–[13]. Many experiments have involved the measurement of just one microwave parameter (e.g., a resonance frequency, the magnitude of a reflected or transmitted wave, etc.) from which can be inferred one piece of information about the sample. Other experiments have measured both the amplitude and phase of a reflected or transmitted wave and, consequently, have the potential of measuring two pieces of information about the sample [14]–[16]. However, this latter case usually requires calculating the impedance (reflection coefficient) or transmittance of an obstacle of specified geometry and arbitrary electrical properties. The configuration of an inductive post in dominant mode rectangular waveguide has been solved theoretically [17], [6] and has been used in recent experiments [8], [9], [12]. However, in order for the inductive post configuration to yield accurate results, its impedance must not be too near the center, or the periphery, of the Smith Chart (i.e., the reflection coefficient should not have a magnitude close to zero or unity). For the case of semiconductor and semimetal materials in which the conduction current is greater than the lattice displacement current, it can be shown [6] that the inductive post should have a radius approximately equal to the classical skin depth. This requirement leads to very small post radii in experiments involving high conductivity materials at the higher microwave frequencies.

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This report discusses an alternative approach to the basic problem involving microwave measurements of high conductivity materials. It will be shown that if such a high conductivity material is used to replace one side wall of a rectangular waveguide, the electrical properties of the material can be computed from the measured propagation characteristics (i.e., the attenuation length and guide wavelength) of this composite waveguide structure. The advantages of this "side wall method" arise not only from the large samples that can be used, but also from the ease of fabricating a planar sample compared to a cylindrical inductive post of small radius and the applicability of this method to the growing technology of thin films. The side wall can easily be capacitively coupled to the metallic waveguide [12], thus avoiding the necessity of making contacts and/or interrupting the microwave current flow [8].

A theoretical study of the characteristics of a perfectly conducting waveguide with a semiconductor side wall is discussed in this paper. It will be shown that in the limit of high conductivity materials, this analysis leads to relatively simple equations that can be interpreted physically. The analytical results are illustrated by an evaluation of the attenuation length and guide wavelength of an RG 138/U waveguide (90 to 140 Gc/s) operating at a frequency of 110 Gc/s with one side wall replaced with indium antimonide with arbitrary electrical properties.

Formulation of the Problem

The basic problem to be discussed involves the characteristics of an electromagnetic wave confined to the interior of a rectangular waveguide bounded on three sides by perfectly conducting walls and on the fourth side by a wall of semiconductor (or semimetal) material of finite conductivity. It will be assumed that the semiconductor wall is sufficiently thick so that there will be a negligible microwave electric field at its exterior surface (i.e., it will be assumed to be many classical skin depths thick). Consequently, the problem involves the determination of the microwave electric field in two regions (the semiconductor and the dielectric filling the interior of the waveguide) with an abrupt planar discontinuity between them.

The semiconductor material comprising the nonmetallic wall will be assumed to be extrinsic or, in the case where both signs of mobile charge carriers are present, that one carrier can be considered relatively immobile by virtue of its low mobility. It will be assumed that the electrical properties of the semiconductor are not functions of position (i.e., that the material is homogeneous and that negligible space charge is present), or functions of the magnitude of the microwave electric and/or magnetic fields [11]. The microwave magnetic field B_1 will be assumed to be small (i.e., $\mu B_1 \ll 1$, where μ is the mobility of the dominant charge in the semiconductor in MKS units).

A differential equation for the microwave electric field within the semiconductor can be derived following the procedure outlined in Larrabee [6]. In this way it is found that if the above assumptions are valid, the microwave electric field in the semiconductor satisfies the following vector relationship (in MKS units)

$$\nabla^2 \boldsymbol{E}_1 + K^2 \boldsymbol{E}_1 = 0 \tag{1}$$

where

$$K^2 = \omega^2 \epsilon \mu_0 + \frac{-j\omega\mu_0\sigma_{AC}}{1 + j\omega\tau} \tag{2}$$

and

$\boldsymbol{E}_1$ = the microwave electric field at any point in the semiconductor,

ω = the radian frequency of the assumed sinusoidal microwave oscillations of the form $e^{j\omega t}$,

ϵ = the dielectric constant of the semiconductor lattice,

μ_0 = the permeability of free space (the semiconductor is assumed to be nonmagnetic),

$j = (-1)^{1/2}$,

σ_{AC} = the conductivity of the charge carriers in the semiconductor at the radian frequency ω and is assumed to be real (i.e., not complex),

τ = a parameter with the dimensions of time that is equal to $\sigma_{AC}\ m/Ne^2$ where m and n are the effective mass and density of the charge carriers, respectively, and e is the electronic charge.

In some cases, τ will be equal to the carrier scattering time. Notice that the semiconductor has been characterized by three electrical properties (ϵ, σ_{AC}, and τ). The purpose of the present analysis is to compute the properties of a waveguide with one semiconductor wall in terms of these three material properties.

The microwave electric field in the dielectric material filling the waveguide will also obey (1) and (2). However, it will be assumed that this material is a lossless dielectric with zero conductivity and a real dielectric constant equal to ϵ^d.

Solution for Transverse Electric Modes

It is now necessary to solve (1) in the two regions of the problem and to apply the appropriate boundary conditions and thus obtain the desired solution. The following discussion will be concerned with only TE_{NO} modes since these modes yield particularly simple equations and include the dominant TE_{10} mode most often used in rectangular waveguide. The Y direction of a rectangular coordinate system will be taken parallel to the electric field of the assumed TE_{NO} mode, and the Z direction will be taken as that normal to the semiconductor-dielectric interface. The microwave electric fields in the semiconductor and dielectric are assumed to have the form

$$E_y^s = A e^{j\omega t} e^{\gamma_x{}^s X} e^{\gamma_z{}^s Z} \tag{3}$$

$$E_y^d = B e^{j\omega t} e^{\gamma_x{}^d X} e^{\gamma_z{}^d Z} \tag{4}$$

where the superscript s refers to the *s*emiconductor and the superscript d to the *d*ielectric. The subscripts x, y, and z refer to the coordinate axes. Thus E_y^s and E_y^d are the microwave electric fields in the y direction in the semiconductor and dielectric, respectively. A and B are complex constants and the gammas are propagation constants as defined by their subscripts and superscripts.

By substituting (3) and (4) into the original differential equations [(1) and (2), which are used as written above for the semiconductor region, and with $\sigma_A = 0$ and ϵ replaced by ϵ^d fir the *d*ielectric region] and applying the appropriate boundary conditions, one obtains[1]

$$e^{2ja(\gamma_x{}^2 + \omega^2\epsilon^d\mu_0)^{1/2}} = -\frac{1 - \left\{1 + \dfrac{(K^s)^2 - \omega^2\epsilon^d\mu_0}{\gamma_x{}^2 + \omega^2\epsilon^d\mu_0}\right\}^{1/2}}{1 + \left\{1 + \dfrac{(K^s)^2 - \omega^2\epsilon^d\mu_0}{\gamma_x{}^2 + \omega^2\epsilon^d\mu_0}\right\}^{1/2}} \tag{5}$$

where

a = the waveguide width (i.e., the distance between the semiconductor-dielectric interface and the opposite conducting wall),

K^s = the value of K as defined by (2) for the *s*emiconductor.

Since the boundary conditions demand that $\gamma_x{}^s = \gamma_x{}^d$, these superscripts have been dropped and both of these quantities are designated as γ_x.

Consequently, this equation can be solved for γ_x and, in this way, determine the propagation characteristics of a waveguide with a semiconductor side wall. However, (5) is a complicated transcendental equation that is not easily handled. Therefore, it is convenient to make an additional assumption which will eliminate the exponential dependence in this equation and facilitate further study of this solution.

Large Conductivity Approximation

If σ_{AC} is large, then the right-hand side of (5) approaches unity, and the exponent of the term on the left-hand side of (5) approaches $2N\pi$, where N is an integer (mode index). If, in fact, the conductivity of the semiconductor were taken as infinite, this exponent

[1] The method ot transverse resonance outlined on p. 389 of Marcuvitz [17] can also be used to solve this problem.

would exactly equal $2N\pi$, and (5) would reduce to

$$\gamma_x^2 = \left(\frac{N\pi}{a}\right)^2 - \omega^2\epsilon^d\mu_0 \tag{6}$$

which is exactly the dispersion relation of a TE_{NO} mode in lossless rectangular waveguide.

If the conductivity of the semiconductor is large but finite, one can simplify (5) by expanding both sides in a power series and retaining only terms to first order in $(K^s)^2-\omega^2\epsilon^a\mu_0)^{-1}$. In this way, one obtains

$$\gamma_x^2 = \frac{N^2\pi^2}{a^2}\left[1 + j\frac{2}{ak}\right] - \omega^2\epsilon^d\mu_0 \tag{7}$$

where for simplicity of notation the quantity $(K^s)^2-\omega^2\epsilon^d\mu_0$ has been called k. A critical review of the several assumptions made in the derivation of (7) reveals that this equation is valid if

$$|ak| > 2N\pi. \tag{8}$$

For convenience of notation, the real and imaginary parts of k will be called k' and k'', respectively, (i.e., $k=(K^s)^2-\omega^2\epsilon^d\mu_0=k'+jk''$, where k' and k'' are real quantities). The propagation constant γ_x can be divided into real and imaginary parts as follows:

$$\gamma_x = \alpha + i\frac{2\pi}{\lambda_g}$$

where α is the attenuation constant and λ_g is the guide wavelength. By using these relationships, (7), and the approximation of (8), one finds that for materials with a positive conductivity (i.e., no microwave generating mechanisms), the complex quantity k must lie in the fourth quadrant (i.e., k' is positive and k'' is negative) and that α and λ_g can be expressed as follows:

$$\frac{1}{\alpha} = \frac{2}{N^2\pi}\frac{a^3}{\lambda_g^0}\frac{(k')^2+(k'')^2}{k'} \tag{9}$$

$$\lambda_g = \lambda_g^0\left[1 + \frac{N^2}{4}\frac{(\lambda_g^0)^2}{a^3}\frac{k''}{(k')^2+(k'')^2}\right] \tag{10}$$

where λ_g^0 is the guide wavelength corresponding to the infinite conductivity case given by (6).

One can compute Poynting's vector for the portion of the solution within the dielectric and thus the power flow in the X direction. For the present case of high-conductivity semiconductors, one can usually neglect the small amount of power that is propagated in the X direction within the semiconductor side wall. In this way, one can compute the voltage-power characteristic impedance Z_0 of the composite structure which, by using the approximation of (8) can be written in the form

$$Z_0 = \frac{2b}{a}\frac{\lambda_g^0}{\lambda_0}\sqrt{\frac{\mu_0}{\epsilon^d}}\left[1 + \frac{k''}{(k')^2+(k'')^2} \cdot \left(\frac{1}{a}\right)\left(1 + \frac{N^2}{4}\left(\frac{\lambda_g^0}{a}\right)^2\right)\right] \tag{11}$$

where b is the height of the waveguide (i.e., the dimension of the waveguide in the Y direction), and λ_0 is the free space wavelength.

The guide wavelength (10) and the characteristic impedance (11) can be interpreted in comparatively simple terms. The penetration of the microwave electric field into the semiconductor side wall makes the waveguide have an effective width slightly larger than the dimension a. One can consider, therefore, replacing the semiconductor side wall with a perfectly conducting side wall that is located a small distance Δ behind the actual position of the semiconductor-dielectric interface so as to make the width of the dielectric in this new waveguide equal to $(a+\Delta)$. The distance Δ can be chosen so as to make the guide wavelength of this perfectly conducting waveguide equal to that of the waveguide with the semiconductor side wall. In this way, one finds that

$$\Delta = -\frac{k''}{(k')^2+(k'')^2}\cdot \tag{12}$$

It is also significant that the characteristic impedance of the perfectly conducting waveguide of width $(a+\Delta)$ is identical with (11). Therefore, one can consider that the change in guide wavelength and characteristic impedance, due to the replacement of the usual perfectly conducting wall by a semiconducting wall, results from an increase in the effective widths of the waveguide by an amount given by (12).

Application of Results to Indium Antimonide

The above equations are best illustrated by a numerical evaluation of (9) and (10) for a specific case. Consider an RG 138/U coin silver ("perfect conductor") waveguide with inside dimensions of 40 by 80 mils operating at a frequency of 110 Gc/s, with $N=1$ (dominant mode). Suppose that one side wall of this waveguide was removed and replaced with a slab of indium antimonide ($\epsilon=18.7\ \epsilon_0$ in MKS units [18]) and that the interior of the resulting structure is filled with air ($\epsilon^d=\epsilon_0$). Figure 1 illustrates the results of an evaluation of (9) and (10) for this particular case. The reciprocal attenuation constant $1/\alpha$ and the change in guide wavelength [i.e., $(\lambda_g^0-\lambda_g)/\lambda_g^0$] are plotted in this figure for various values of the conductivity of the indium antimonide and for various values of the parameter τ (expressed as $\omega\tau$). Notice that at low values of $\omega\tau$ and high values of conductivity, the curves of Fig. 1 are nearly orthogonal and inclined at an angle of approximately 45° to the axes of the graph. In this region, one can use these curves [or (9) and (10)] to convert from the semiconductor properties σ_{AC} and τ to the waveguide properties α and λ_g, or vice versa, with a high degree of accuracy. Notice, however, that at low conductivities and/or high $\omega\tau$ values, the curves of Fig. 1 lose their near orthogonal characteristics, and their value in converting from the semiconductor to the waveguide properties diminishes.

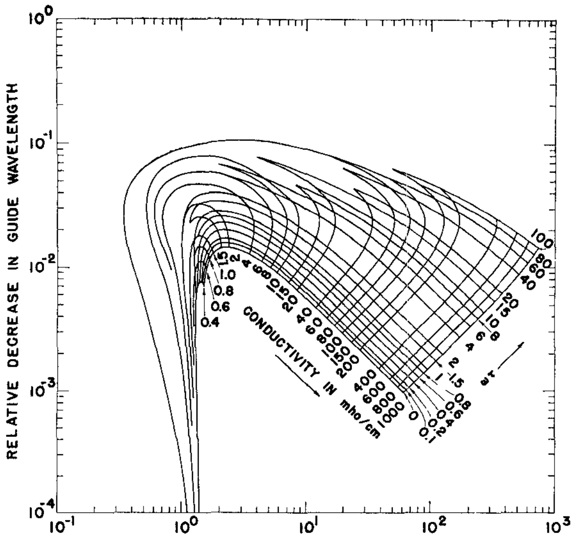

Fig. 1. The theoretical attenuation and guide wavelength of an RG 138/U waveguide (inside dimensions 40 by 80 mils) operating in the TE_{10} mode at 110 Gc/s. One side wall of this waveguide has been replaced with indium antimonide with the electrical properties shown in the figure. The relative dielectric constant of the indium antimonide was assumed to equal 18.7, and that of the air filling the interior of the guide equal to unity. The distance along the waveguide required to decrease the voltage of the propagating wave by a factor of e is plotted horizontally (multiply this scale by 0.346 to convert to the distance required to halve the power in the wave). The guide wavelength is expressed along the vertical scale of this figure as the relative decrease in guide wavelength caused by replacing a perfectly conducting side wall by the semiconductor side wall. This figure was obtained by plotting the results of a numerical evaluation of (9) and (10).

The curves of Fig. 1 were computed on the assumption that the absolute value of ka was greater than 2π [see (8)]. Figure 2 represents a graph of the contours of equal value of $|ka|$ as computed from the above approximate equations as a function of the semiconductor parameters. The region near the contour, with a value of ten represents a region in which there is a partial cancellation of the inductive component of the motion of the semiconductor charge carriers by the capacitive displacement current of the semiconductor lattice. Therefore, since one is generally interested in measuring effects due to the motion of the charge carriers (i.e., σ_{AC} and τ), one might expect to find the largest dependence upon these parameters in the regions of Fig. 2 corresponding to a larger value of $|ka|$ than this minimum. Figure 3 shows that this is indeed the case, for it shows that the portions of the curves of Fig. 1 that are the most useful correspond to the high $|ka|$ values in the lower right-hand side of Fig. 2. It is apparent that the present high-conductivity assumption [i.e., (8)] is a self-consistent assumption in those regions of the curves of Fig. 1 [or (9) and (10)] that are most useful for computational purposes.

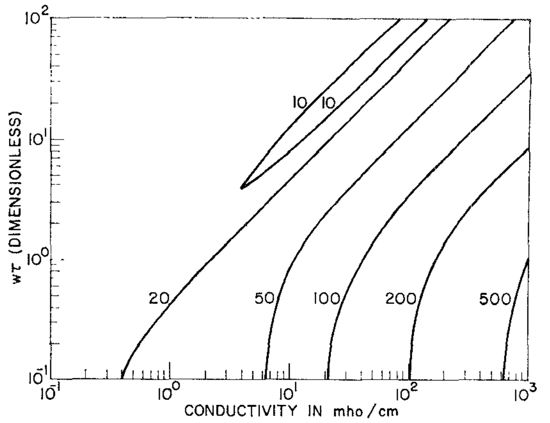

Fig. 2. A graphical illustration of the contours of equal value of the parameter $|ka|$ as a function of the electrical properties of the indium antimonide with the conditions of Fig. 1. At large conductivities and small values of $\omega\tau$, this parameter is indeed larger than 2π, as was assumed in the derivation of (9) and (10).

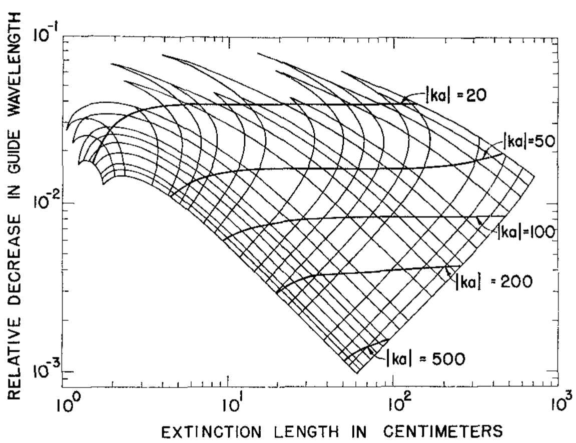

Fig. 3. A graphical illustration of the contours of equal value of the parameter $|ka|$ of Fig. 2 superimposed on the curves of Fig. 1. Notice that in the region where the curves of Fig. 1 show a large dependence on the electrical properties of the indium antimonide, the parameter $|ka|$ is much larger than 2π, thus illustrating the self-consistency of this assumption in this region of the curves of Fig. 1.

Conclusions

The attenuation, guide wavelength, and characteristic impedance of waveguides propagating TE_{NO} modes with one side wall replaced by a high-conductivity semiconductor have been derived. It has been shown that the penetration of the microwave electric field into the semiconductor material effectively increases the width of the waveguide by an amount of the same order as the classical skin depth. The analytical results have been evaluated for the case of an indium antimonide side wall in RG 138/U waveguide operating at 110 Gc/s. The calculated attenuation lengths and guide wavelengths for this case are of such magnitude that they can be measured with reasonable accuracy, thus illustrating the value of this technique for the measurement of the high-frequency properties of indium antimonide.

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A Technique for Measuring Individual Modes Propagating in Overmoded Waveguide

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***Abstract*—A practical measurement technique for determining the relative amplitude and phase of the individual modes propagating in overmoded waveguide is described. A phase-sensitive detector is used to measure the output of fixed probes placed around a single transverse plane in a section of enlarged waveguide. The detected output is directly proportional to the modal components, and data reduction is performed manually. The use of oversize waveguide provides increased accuracy and permits total multimode power measurements in conjunction with mode analysis. The technique can be used for mode measurements up to the fourth harmonic in standard rectangular waveguide. Experiments described in the paper use a single frequency source. However, signal sources with spurious content can be evaluated using appropriate tunable RF band-pass filters.**

Introduction

MEASUREMENTS in waveguide containing two or more propagating modes has received considerable attention from microwave engineers in recent years. This paper describes a practical technique for mode measurement that can provide rapid performance data on waveguide components subjected to overmoded propagation of power.

An effective device or technique for measuring multimode power must be able to determine the power in each mode or measure the total power regardless of the number of modes present. Both approaches have been used by various independent investigators to measure multimode power, and a number of techniques have been developed for use with rectangular waveguide [1]–[6]. Of particular interest here is the technique reported by Taub, based on the use of an enlarged section of waveguide [1]. This technique measures the total multimode power in the waveguide without identification of, or regard for, the individual propagating modes.

This paper describes a simple and practical measurement technique for identifying the individual modes and determining their relative amplitude and phase using the same oversize waveguide as Taub. Whereas Taub's method requires enlarged waveguide as a basic component, its use for mode measurements proves to be advantageous from practical considerations rather than being required by theoretical considerations.

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